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INFLUENCE OF ROTOR BLADES ON CONVERGENT-DIVERGENT NOZZLE AERODYNAMICS AND OVERALL SUPERSONIC TURBINE

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ABSTRACT

Single-stage axial pressure compound and two-row velocity compound supersonic turbines have a very high specific work, which allows for high power levels at high-pressure ratios and small flow rates in a very compact dimensional envelope. However, despite their mechanical simplicity, the supersonic aerodynamic effects in these turbines are very complex. There are many design and analysis methodologies for these turbines, which are currently used in the industry. Nevertheless, several important parameters are still insufficiently investigated. In an earlier paper (GT2022-83387), the authors considered the influence of convergent-divergent nozzle arrangement parameters on aerodynamic losses and overall supersonic turbine performance, which partially closed the knowledge gap. However, the flow at the nozzle ring's exit is influenced by the rotor blades, especially by the geometry of the blades leading edge; axial clearance between the nozzle and rotor cascades; and a combination of other nozzle arrangement parameters at different Mach numbers. This is not well studied and needs to be further investigated, which will be the focus of this paper.

This paper examines the qualitative and quantitative influence on aerodynamic losses of the rotor blade, such as the leading edge geometry; axial clearance; and meridional flow angle at the exit of convergent-divergent conical nozzles. The methods utilized to determine the influence of these parameters are provided and the results are discussed. An approach to incorporate the quantitative corrections of the nozzle loss models is presented and a comparison with experimental data is performed. The incorporated models allow for designing and analyzing the supersonic turbines considering more parameters enabling the opportunity to fine-tune the design to specific requirements with higher confidence and accuracy.

Keywords: Supersonic Turbine, Convergent-Divergent Nozzles, Rotor Blades, Turbine Performance, Aerodynamic Losses, Turbine Design, Turbine Analysis

NOMENCLATURE

α	- is an angle taken by the length of the exit ellipsis major axis (H) of a nozzle placed at the mean diameter $\mathrm{D_{mean}}$
α'	- the angle between centers of exit ellipses of two neighboring nozzles
H	- the major axis of the outlet ellipse of the nozzle
D_{mean}	- mean diameter at the outlet of the nozzle
D_{AB}	- diameter of a circle whose center coincides with the center of a circle with a diameter $\mathrm{D_{mean}}$ and which passes through the points of contact of the main axis of the nozzle outlet ellipse with the outlet ellipse.
L	- the length of the arc of a circle with a diameter $\mathrm{D_{mean}}$ that is located inside the outlet ellipse of the nozzle.
U/C_0	- isentropic velocity ratio
φ	- velocity coefficient of the nozzle
ψ	- velocity coefficient of the rotor
γ	- isentropic exponent
FMM	- formal macromodel
PR	- total-static pressure ratio
i	- point number.
n	- a total number of points.
$deviation$	- nozzle loss deviation at the current point
$MIDDEV$	- nozzle loss average deviation

1. INTRODUCTION

Supersonic axial turbines most often have a single-stage single-row impulse or single-stage combined speed stage with two or more rows of rotor blades [1] and have a very high specific work. This makes it possible to achieve high power levels at high-pressure drops and low flows in a very compact package. The latter is highly desirable in some types of liquid propellant rocket engine (LPRE) turbopumps. Due to their compactness, mechanical simplicity, and low cost, these turbines are also suitable for organic Rankine cycles (ORC) [2, 3] with high-pressure ratio, gas expanders in chemical and technological processes, various mechanical drives, etc. [1, 4].

One of the main elements of this type of turbine, affecting the overall efficiency, is the first nozzle apparatus. Therefore, considerable attention in the literature is paid to experimental [5] and theoretical studies of fluid flow processes in supersonic nozzles [5, 6] and methods for their profiling, starting from the two-dimensional theory [8, 9] and ending with the design of real structures of nozzle cascades [8, 7]. The velocity coefficient of the nozzles has a greater influence on the turbine stage operation than the velocity coefficient of the rotor blades. An increase in the velocity coefficient of the nozzle apparatus by 1% corresponds to an increase in the internal efficiency by 1-2.5% [10, 11, and 12]. In [11] it was shown that an increase in φ by 1% corresponds to an increase in the circumferential efficiency of a stage by 2%. In [12] the authors determined that an increase in φ by 1% corresponds to an increase in internal efficiency up to 2.5%. For air microturbines [11], the influence of the direct velocity coefficient of the rotor ψ on the efficiency of the step wheel is 3-4 times weaker than the influence of the nozzle velocity coefficient φ. Drilled nozzles are often used in supersonic turbines. This is a conventional name in the turbomachinery field for the nozzles with a circular cross-section of the throat and divergent parts of the nozzle because they are typically manufactured by drilling and reaming processes. Nozzles of this type do not have secondary losses and have relatively small losses in the boundary layer due to the small wetted perimeter of the circular channel [13]. In this regard, they are widely used in turbines with a partial admission inlet of the working fluid [14]. The total losses in the nozzle cascade, which consists of similar nozzles, are influenced by several factors that also have a mutual influence on each other. Conventionally, they can be divided into several types of losses.

One of these factors is the geometry of the nozzle itself, which affects both the nozzle flow rate and the internal energy loss in the nozzle [15]. Nozzles with profiled (contour) expanding sections have a slightly higher efficiency compared to nozzles with conical expanding sections, but in operating modes with a pressure drop below the calculated efficiency of the latter is sometimes better [16].

The location of the nozzles relative to each other with a cascade also has an influence [10, 18-20]. The exit section of the drilled nozzles has an elliptical shape, which leads to a greater non-uniformity of the flow at the inlet to the rotor blades, compared to grids with prismatic nozzle blades. With partial admission of the working fluid, several drilled nozzles are usually placed in the arc section of the inlet, often called the “nozzle ring”, while the supersonic flow going from one nozzle interacts with the flow from neighboring nozzles, which leads to additional losses [14]. The effect of flows from neighboring nozzles on the efficiency of their operation and the overall efficiency of the turbine has been studied by many authors [10, 18, 19].

Previously, the authors conducted a study of the Influence of the convergent-divergent nozzles arrangement parameters on aerodynamic losses and overall supersonic turbine performance [20]. Nozzle cascades with conical nozzles were considered. As a result of the study, the model of aerodynamic losses of conical drilled nozzles was improved by including a coefficient that depends on such parameters of the nozzle ring as the drilling angle, the relative pitch, and angle of the nozzle, the relative size of the nozzle [20].

The total losses in the nozzle cascade are influenced by the rotor cascade, in particular, the leading edge of the rotor blade, on which a shock takes place, as well as the axial gap between the nozzle and rotor cascade. In the paper [21], a study was carried out to improve the flow pattern in the turbine stage associated with the axial gap between the stator and the rotor. This study shows that improvement of the flow pattern between the nozzle exit-rotor inlet gap has an essential role in optimizing impulse turbine with a partial admission. Such an optimization results in a reduction in the total pressure losses and a decrease in the sensitivity of the nozzle to the expansion rate. In the paper [22] the influence of the effects of axial gap and nozzle distribution on aerodynamic forces of a supersonic partial-admission turbine was considered. The study was conducted on a 2-stage turbine in an LH2/LOX rocket engine. It has been shown that the turbine efficiency decreases as the axial gap between the nozzles and the 1st stage rotor increases.

The papers [23, 24] carried out a study of rotor leading edge shape on performance of supersonic axial impulse turbine. The paper contains the investigation of the rotor blade sweep and hub end wall contouring effects on the performance of small-scaled supersonic axial impulse turbines. The mathematical formulation of the rotor blade sweep and radial velocity component at the rotor inlet was proposed. The utilization of backward swept rotor blades together with the positive lean provided efficiency increasing up to 2.9%.

Failure to account for the influence of these parameters can lead to discrepancies between predicted and actual performance and thus additional design iterations, increasing turbine development time and cost. In the study presented in this article, only turbine stages with conical drilled nozzles are considered.

2. MODEL FOR ACCOUNTING OF THE INFLUENCES OF THE ELEMENTS OF THE TURBINE STAGE, INDIVIDUAL NOZZLE GEOMETRY, AND THEIR LOCATION IN THE CASCADE

An earlier paper (GT2022-83387) [20] presented a model that takes into account internal nozzle loss and the effect of converging and expanding nozzle arrangements on aerodynamic losses and the overall performance of a supersonic turbine.

FIGURE 1 shows the sketch of a parametric model of a conical drilled nozzle with the respective eight geometrical parameters unequivocally defining its shape and two parameters that characterize the geometry of the rotor blade and the turbine stage (thickness of the rotor leading edge and axial clearance between the nozzle and rotor cascades).

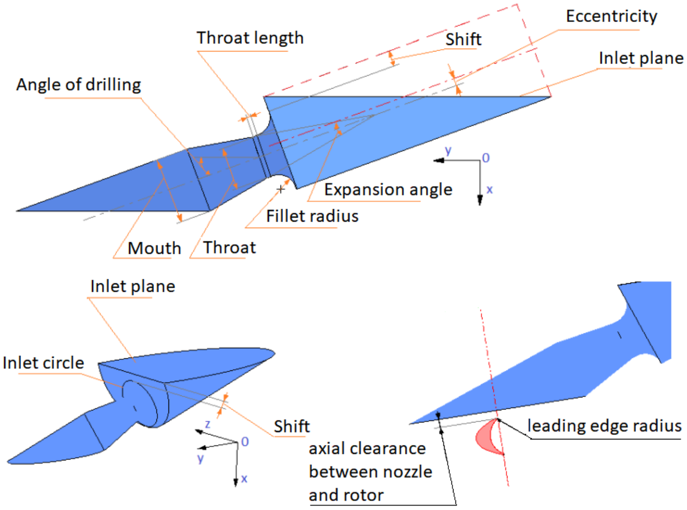


FIGURE 1: SKETCH OF A CONICAL DRILLED NOZZLE PARAMETRIC MODEL

The developed loss model for individual nozzles $\varsigma_{individual\ nozzle}$ was used in the current study as the base, i.e. the aerodynamic losses associated with nozzle arrangement parameters were introduced as the multiplication coefficients $k_{nozzles\ arrangement}$ and $\boldsymbol{k_{infl\_stage}}$ so the total aerodynamic losses in the nozzle ring $\varsigma_{nozzle\ ring}$ can be determined by the following equation:

$$\varsigma_{nozzle\ ring} = \varsigma_{individual\ nozzle} \cdot \boldsymbol{k_{nozzles\ arrangement}} \cdot \boldsymbol{k_{infl\_stage}}$$

- $\varsigma_{individual\ nozzle}$ is a function of Throat, Throat length ratio, Fillet radius ratio, Shift ratio, Eccentricity ratio, XPR, Expansion angle, and Angle of drilling.

Conical drilled nozzle geometrical parameters: angle of drilling (the angle between the axis of the nozzle and a plane parallel to the exit ellipse of the nozzle), throat length (the length of the cylindrical part of the nozzle from the throat section to the conical part of the nozzle), fillet radius (the radius that sets the surface of revolution at the nozzle inlet from the inlet circle to the throat section), expansion angle (the angle that defines the conical surface of revolution from the throat to the mouth (its choice determines the length of the nozzle)), throat (diameter of a circle at the inlet of the conical part of the nozzle), mouth (diameter of a circle at the outlet of the conical part of the nozzle), shift (the distance between the inlet circle and the inlet plane of the nozzle). eccentricity (the distance between the nozzle axes and the axis of the cylindrical surface of the nozzle inlet section (red dotted line), this surface always passes along the tangent to the nozzle inlet circle, and the shape of the nozzle inlet part depends on this value).

It was critical making the aerodynamic loss model for individual nozzles applicable to the vast majority of working fluids. This was achieved based on the fact that the Reynolds number and Mach number are the most important parameters determining similarity for supersonic flows. Therefore, when the model for the internal aerodynamic losses for individual nozzles was under development the ranges for variation of Throat and XPR were selected to keep Reynolds number (calculated using Throat as the specific size) and exit Mach number in typical ranges for various fluids. The loss model for an individual nozzle was developed for a range of theoretical Mach numbers from 1.9 to 4. For the Reynolds number, the range is from $2 \cdot 10^5$ and above.

- $\boldsymbol{k_{nozzles\ arrangement}}$ is a function of the Angle of drilling, Relative pitch, Nozzle angle, and Turbine mean diameter factor (FIGURE 1, FIGURE 2).

Relative pitch - The relative pitch for the arrangement of drilled nozzles is defined as the ratio of the angle (α') between centers of exit ellipses of two neighboring nozzles to the angle (α) taken by the length of the exit ellipsis major axis (H) of a nozzle placed at the mean diameter D_{mean} of the turbine

Nozzle angle - is an angle between the major axis of the nozzle exit ellipsis and the tangent to the circle of mean diameter at the point where the center of the exit ellipsis touches it

Nozzle relative size - is the ratio (H/L) of the length of the major axis of the ellipse H to the length of the arc L of a circle of diameter D_{AB} that passes through points A and B, with the same center 0 as the circle of mean diameter.

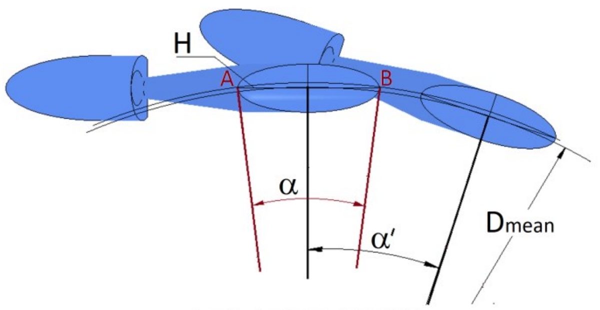
- $\boldsymbol{k_{infl\_stage}}$ is a function of the Rotor blade leading edge radius, Axial clearance between nozzle and rotor cascade, Nozzle angle, and XPR (FIGURE 1, FIGURE 2).

The rotor blade leading edge radius - is a parameter that determines the thickness of the leading edge of the rotor blade.

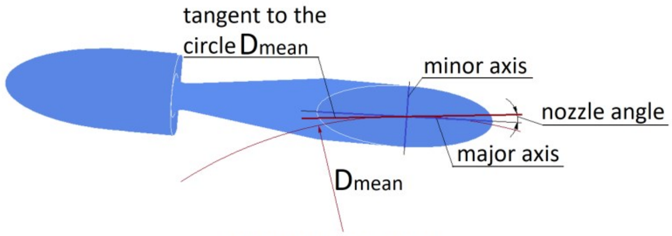
Axial clearance between nozzle and rotor cascade - this is a parameter that determines the value of the axial clearance between the nozzle and rotor cascade.

XPR - is defined as the ratio of the area of the circle at the exit of the conical section (Mouth) to the area of the circle at the inlet to the conical section (Throat – diameter of a circle in a given section, the narrowest part of the nozzle). Essentially XPR is a squared ratio of Mouth to Throat.

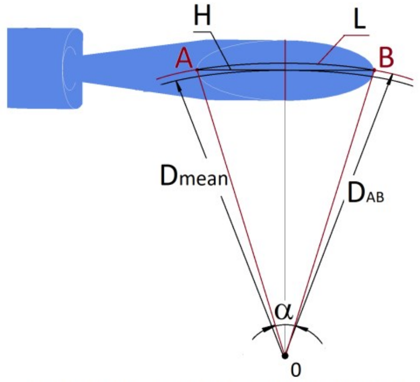
The binding of loss levels of three series of calculations: calculations of the influence of geometry on the internal losses of individual nozzles, the influence of the arrangement of nozzles in cascade, and the influence of stage elements was made based on the selected reference geometry of the nozzle. That is, a nozzle with this geometry was used in the entire series of studies. This allowed for a specific nozzle geometry to separate the influence of the location of nozzles with a nozzle cascade ($\boldsymbol{k_{nozzles\ arrangement}}$). And in a further series of calculations to separate the influence of the elements of the stage ($\boldsymbol{k_{infl\_stage}}$).



RELATIVE PITCH



NOZZLE ANGLE



TURBINE NOZZLE RELATIVE SIZE

FIGURE 2: TO THE DEFINITION PARAMETERS OF THE TURBINE NOZZLE CASCADE

3. CFD CALCULATIONS PARAMETERS

The computational studies of the variants of the turbine according to the Box-Behnken plan (TABLE 1) were carried out using the ANSYS CFX software package.

The computational domains were a turbine with a nozzle and rotor cascades (FIGURE 3). The computational domain included a nozzle cascade in which there were 3 nozzles. The nozzles were combined into a nozzle box at the inlet into which the total parameters of the working fluid were set. The rotor cascade at the root and peripheral ends had stationary walls. The rotor blade itself is defined as a rotating wall. The type of interface between the stator and the rotor is set - frozen rotor. Static pressure was set at the exit from the rotor domain.

The working fluid was modeled as an ideal gas (isentropic exponent γ=1.4). SST [25] turbulence model was used for all calculations in this study. The parameters for constructing the grid in the computational domains and the boundary layer inside the nozzles and around the rotor blades were chosen based on our experience [20] and the recommendations [25]. For the SST model, wall functions are used, which makes it possible to correctly carry out calculations using the calculated grids with different degrees of thickening as they approach the wall [25]. In this case, the first wall node can fall into one of the following regions of the boundary layer: the logarithmic range ($30 \leq Y+ \leq 100$), into a viscous sublayer ($Y+ \leq 5$), and the transition region between them.

The sizes and number of the mesh cells in the boundary layer for all variants of the computational areas were chosen the same and were chosen to approach the above recommendations. In the current study, Y+ in the computational domain was within a range between 30 and 100 in all calculations.

The boundary conditions ranges in the calculations were as follows: total-static pressure ratio PR = 7 – 45, Mach number =1.9 – 3. The ranges of geometric parameters presented in TABLE 1 are as follows: Leading edge radius 0.01 - 0.5 (ratio to the chord of the rotor), Axial clearance 0 - 2.35 (ratio to the minor axis of the outlet nozzle ellipse), Nozzle angle -8° - +2°, XPR = 1.5 – 4.2.

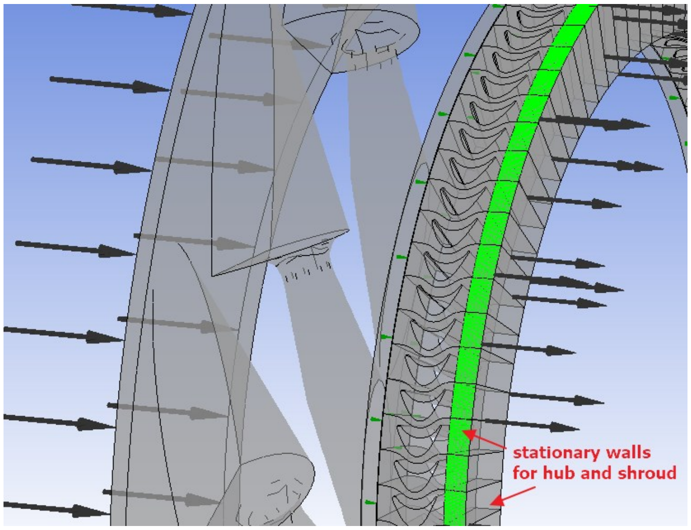


FIGURE 3: COMPUTATIONAL DOMAIN EXAMPLE

Below (FIGURE 4) is an example of a mesh in the computational domain around the rotor blade for two options with different thicknesses of the leading edge of the blade profile. The blade is made in a prismatic shape.

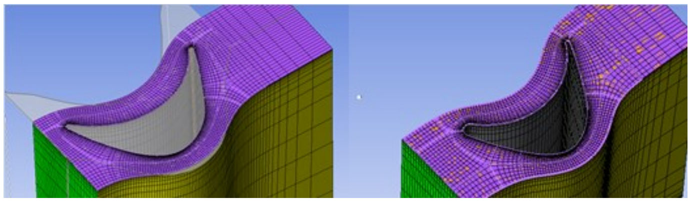


FIGURE 4: EXAMPLE OF A MESH AROUND A ROTOR BLADE PROFILE

FIGURE 5 shows an example of distributions of streamlines and Mach numbers in the computational domain.

As a result of the calculations, the aerodynamic characteristics of the nozzle cascades of different configurations of turbine stages were obtained. In Table 1 the calculated loss values in relative terms are presented. Losses are divided by the minimum obtained value of losses at point 19.

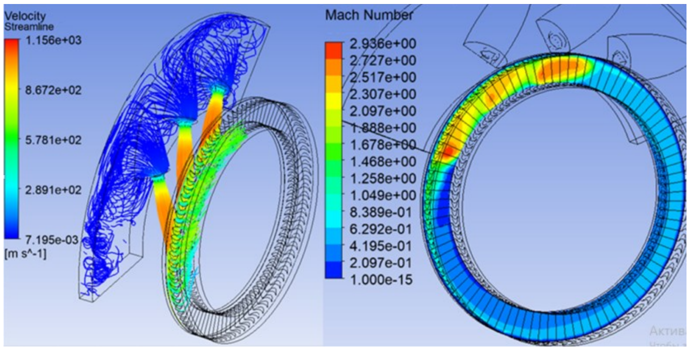


FIGURE 5: EXAMPLE OF DISTRIBUTIONS OF VELOCITY STREAMLINES AND MACH NUMBERS IN THE COMPUTATIONAL DOMAIN

4. FORMAL MACROMODEL AND ACCURACY ANALYSIS

One of the most reliable optimization methods is the method of direct enumeration of discrete values of variables, generated randomly or using some quasi-random sequences, such as the Sobol quasi-random sequence. However, this search method might take a lot of calculation time especially when the objective function is resource-intensive.

One of the ways that allow reducing the number of calculations when solving the optimization problem is to replace the objective function with its approximation or interpolation dependence - a formal macromodel (FMM). In the context of this study, FMM was used as a response surface which reflects the behavior of the nozzle loss correction coefficient.

In [20] a modified Rechtschaftner plan was used to compile a formal macromodel, which made it possible to increase the accuracy of the model but at the same time significantly increased the number of calculation points. In this case, it was necessary to carry out many calculations in a relatively complex and large computational area - the turbine stage. In this regard, the Box-Behnken plan [26] was chosen to compile a formal macromodel to perform computational studies in a reasonable time.

TABLE 1 shows the Box-Behnken plan for four variables (Leading edge radius, Axial clearance, Nozzle angle, XPR), based on the combinations of which a formal macromodel was compiled.

As a result of the calculations, the aerodynamic characteristics of the nozzle rings taking into account the influence of turbine stage elements were obtained. In TABLE 1 the calculated loss values in relative terms are presented.

As a result of the computational studies and combination of CFD results for the Box-Behnken a formal macromodel was built and the response function coefficients were obtained, taking into account their mutual influence on the aerodynamics of the flow behind them.

TABLE 1: BOX-BEHNKEN EXPERIMENT PLAN

N point	Leading edge radius	Axial clearance	Nozzle angle	XPR	Relative losses
1	1	1	0	0	3.76
2	0	0	1	1	2.37
3	1	0	0	1	3.69
4	0	1	1	0	3.00
5	1	0	1	0	3.14
6	0	1	0	1	2.90
7	1	-1	0	0	1.94
8	0	0	1	-1	3.13
9	1	0	0	-1	3.29
10	0	1	-1	0	2.80
11	1	0	-1	0	3.04
12	0	1	0	-1	3.44
13	-1	1	0	0	2.18
14	0	0	-1	1	2.36
15	-1	0	0	1	1.43
16	0	-1	1	0	1.47
17	-1	0	1	0	1.94
18	0	-1	0	1	1.35
19	-1	-1	0	0	1.00
20	0	0	-1	-1	2.81
21	-1	0	0	-1	2.77
22	0	-1	-1	0	1.34
23	-1	0	-1	0	1.72
24	0	-1	0	-1	2.01
25	0	0	0	0	2.17

FIGURE 6 shows a comparison of the deviation of the relative loss FIGURE 1 (relative losses - are divided by the minimum value of the losses), calculated by CFD and predicted by the response function according to the Box-Behnken plan. With that being said, the error is in the range of 20% to 40% for about 15% of all points, for the other 50% of the points the error is in the range of 10% to 20%, and for the rest of the points, the error is between 0% and 10%.

The average deviation of loss values calculated using the following equation at plan points does not exceed 15.3 %.

$$MIDDEV = \frac{\sum_{i=1}^{n} deviation}{n}$$

where

$MIDDEV$ – nozzle cascade loss average deviation,
$deviation$ – nozzle cascade loss average deviation,
i – point number,
n – total number of points.

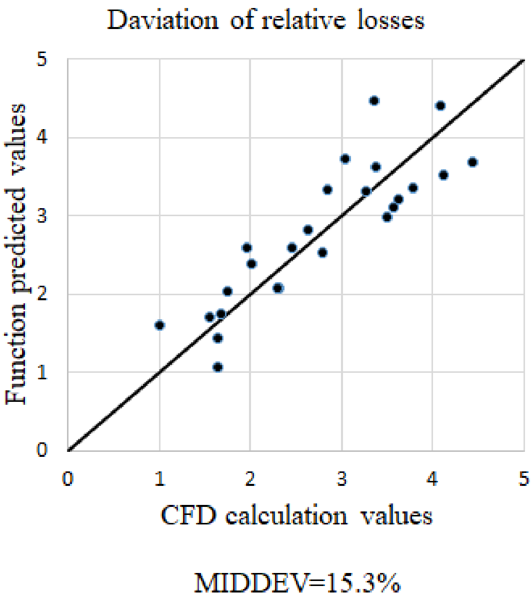


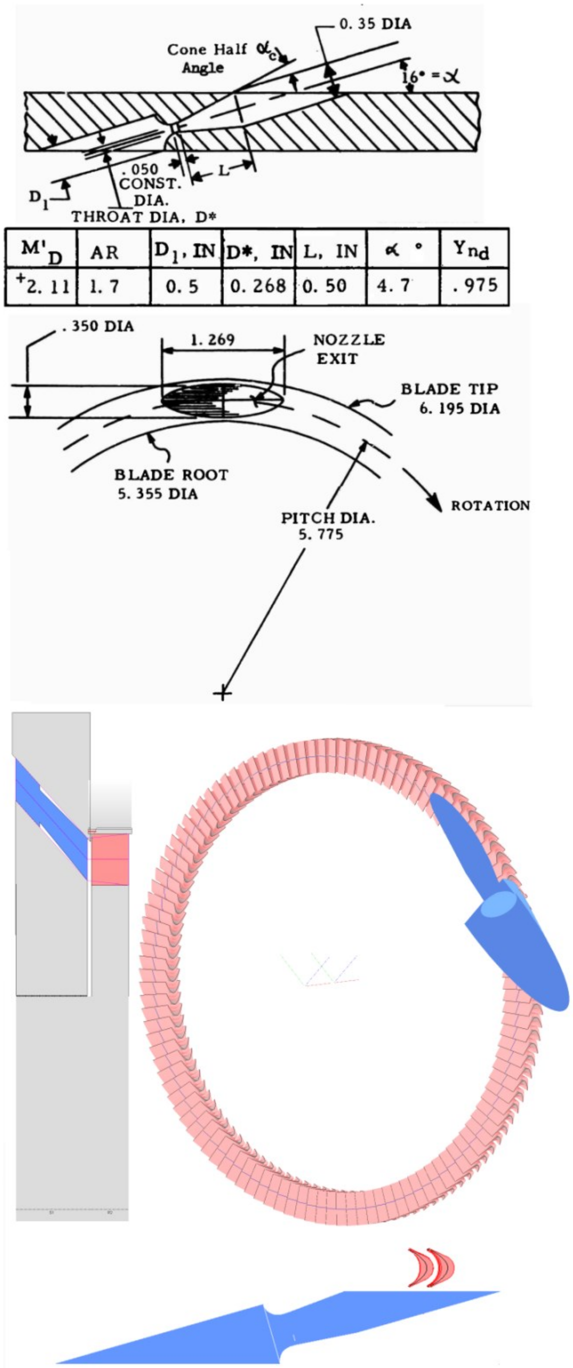
FIGURE 6: DEVIATION OF RELATIVE LOSSES (BOX-BEHNKEN)

5. EVALUATION OF THE MODELS INCORPORATED IN THE TURBINE DESIGN

In order to evaluate how the developed loss model works together with the previously developed loss models and turbine design and analysis tools, it was decided to take the calculated design of the turbine from [20]. Essentially the supersonic turbine with drilled nozzles from [16] was recreated and performance was compared with the one obtained by AxSTREAM® with new loss coefficients.

In [16] various configurations of nozzles were analyzed with respect to their influence on the efficiency of a supersonic experimental turbine. One of the considered nozzles was drilled with a conical divergent section. The nozzle was designed for a Mach number of 2.1. The testing was carried out with nitrogen as a working fluid and a pressure ratio of PR=10. The inlet temperature was 80°F (299.8°K). The drilled nozzle had the following geometrical parameters: 16° angle of drilling, 1.7 XPR, 0.268-inch throat diameter (0.0068 m), 4.7-degree nozzle half-angle (half of the expansion angle). Nozzle velocity coefficient 0.975. The number of rotor blades was 115. Nozzle parameters and basic geometric parameters of the turbine are shown in FIGURE 7. The geometry of this turbine was replicated in AxSTREAM® (FIGURE 7).

FIGURE 8 shows a comparison of calculated and experimental data on turbine efficiency for various U/C$_0$ speed ratios. Experimental turbine efficiency is shown in black and AxSTREAM® [20] predicted efficiency is shown in red. The blue color shows the curve that was obtained using the developed loss system. It can be seen that taking into account the effect on losses in the nozzles of additional parameters that are taken into account in the new system of losses has led to a change like the dependence of the turbine efficiency on U/C$_0$. This made it possible to obtain calculated data closer to the external data.



M'_D	AR	D_1, IN	D*, IN	L, IN	α °	Y_{nd}
+2. 11	1. 7	0. 5	0. 268	0. 50	4. 7	. 975

FIGURE 7: NOZZLE PARAMETERS [16] AND REPLICATED TURBINE DESIGN IN ASTREAM®

The calculation of this turbine was also performed for a pressure ratio of PR=15. A comparison of the calculated and experimental data is shown in FIGURE 9. A fairly good agreement between the calculated and experimental data was obtained.

Estimates were made of the deviations of the calculated data from the experimental points presented in FIGURE 8, and FIGURE 9. The maximum deviation of the calculated data from the experimental points presented in FIGURE 8 is PR=10 (red line) 11.8%, PR=10 (blue line) 10 %, and in FIGURE 9 PR=15 (yellow line) 10.7 %. The average deviation of the calculation values from the experimental points in the case is PR=10 (red line) 4.7% PR=10 (blue line) is 3.15%, and in the case PR=15 (yellow line) is about 3%.

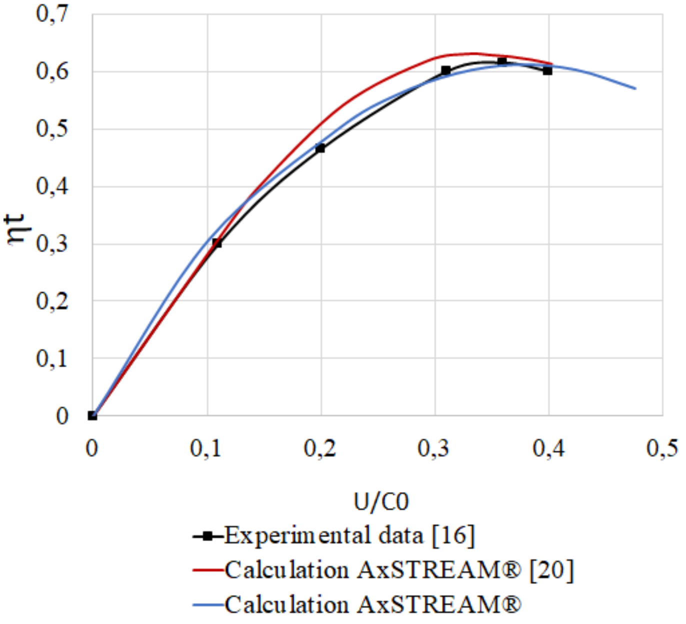


FIGURE 8: COMPARISON OF PREDICTED PERFORMANCE WITH EXPERIMENT (PR=10)[16]

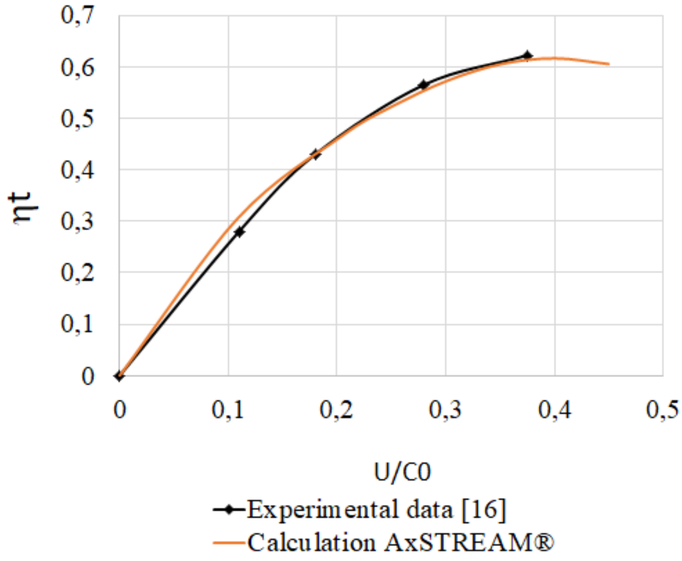


FIGURE 9: COMPARISON OF PREDICTED PERFORMANCE WITH EXPERIMENT (PR=15)[16]

Also, using the developed loss model, a computational study of the influence of the axial clearance and the thickness of the leading edge of the rotor blade on the nozzle velocity coefficient was performed. The turbine presented in this section [16] was taken for research. FIGURE 10 shows the results of this study. Parameter values are presented in relative terms. The nozzle velocity coefficient is related to the maximum calculated value of the velocity coefficient, as can be seen from the figure, this value corresponds to the minimum value of the thickness of the leading edge of the rotor blade and the minimum value of the gap. The value of the thickness of the leading edge is related to the chord of the rotor blade, and the value of the axial clearance is related to the height at the exit from the drilled nozzle (minor axis of the ellipse). It can be seen that as the axial clearance increases, the velocity coefficient of the drilled nozzle decreases. The same thing happens with an increase in the thickness of the leading edge of the nozzle. The results obtained do not contradict the data presented in the literature.

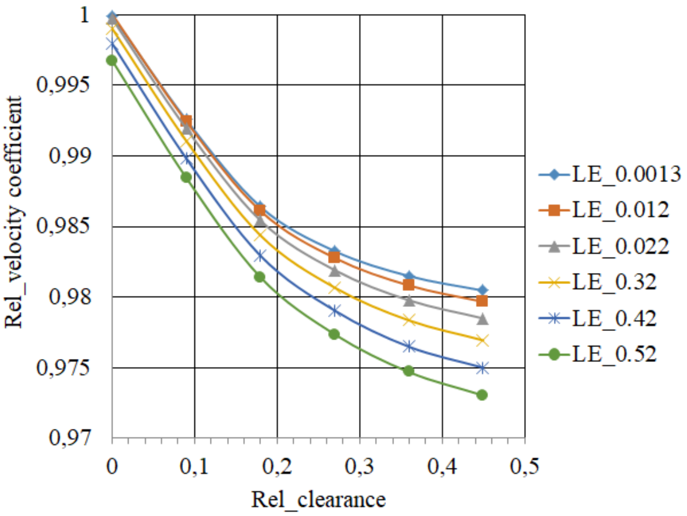


FIGURE 10: INFLUENCE OF THE AXIAL CLEARANCE AND THE THICKNESS OF THE LEADING EDGE OF THE ROTOR BLADE ON THE VELOCITY COEFFICIENT OF THE DRILLED NOZZLE

Thus, the developed loss model makes it possible to predict with satisfactory accuracy the level of velocity coefficients (and, accordingly, losses) in cascades with drilled nozzles.

6. CONCLUSION

As a result of the performed study, the aerodynamic loss model for conical drilled nozzles was improved by incorporating the coefficient which depends on parameters of elements of the turbine stage such as axial clearance and rotor leading edge thickness.

The comparison of the performance predicted by AxSTREAM® with the use of the improved drilled nozzles loss model for two supersonic turbines showed a good agreement.

The utilization of the improved loss model provides the opportunity to design high-efficient supersonic turbines with conical drilled nozzles and satisfy specific design constraints with the use of nozzle arrangement parameters.

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