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PREDICTION OF STRUCTURAL SUPPORTS INFLUENCE ON ROTATING MACHINERY DYNAMICS

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ABSTRACT

Rotor lifetime and safety primarily depend on the level of rotor vibration. In order to avoid unwanted consequences for the plant due to rotor damage and to meet the highest requirements of design reliability, accurate rotor dynamic predictions are mandatory. Having the correct rotor model is a critical issue in dynamics prediction. Often research activities are focused only on the rotor-bearing system analysis. However, generally, the whole system, which includes the rotor, bearings, casing and structural supports should be considered. Special attention should be paid to the influence of structural supports which reveals when the rotor is supported by ball bearings because of low damping and high bearing stiffness.

The approach presented in this paper allows us to simulate the influence of structural supports on rotor dynamics response and as a result, the full picture of rotor-bearing-support system resonances can be analyzed to avoid potential problems. The methodology is based on support vibrations modal reduction technics. According to the approach, the natural frequencies and their mode shapes should be calculated for the separate support structure applying a three-dimensional finite element model and the relative displacements at bearing location points are measured. Supports' normalized modal characteristics (modal mass and modal stiffness) for each vibration mode should then be imported in a rotor dynamics algorithm for rotor unbalance response analysis.

The approach allows for simulation of different types of support structures such as bearing pedestals, steel foundations, tabletop-type foundation, frame and pipe supports of arbitrary geometry, and so on. Validation based on the Jeffcott rotor model is presented. The current methodology has been applied to a single- stage compressor's rotor-bearing-support system which was manufactured and commissioned. The results of the simulations are discussed.

NOMENCLATURE

RBFS – rotor-bearing structural supports system
RBS – rotor-bearing system
FE – finite element
FEA – finite element analysis
DF – dynamic forces
DOF – degree of freedom
MP – modal parameters

INTRODUCTION

Prediction of rotor vibrations is a significant task in rotating machinery design and analysis. Many scientists are involved in rotor dynamics research, and the rotor dynamics analysis theory is presented in various fundamental books [1], [2], [3].

A good practice for safe rotor design typically involves the avoidance of any resonance situation at operating speeds with some margins including resonances by a reason of structural supports vibration. If the design can't be changed to satisfy frequency margins, the vibration amplitudes and forces acting on the bearings have to be measured and analyzed according to standards or company design practice [4].

It is well-known that critical speeds of rotor-bearing systems which are supported by pedestal or steel foundation are different in comparison to separate rotor-bearing systems without supports [5]. New resonance peaks in the rotor amplitude-frequency response characteristics also appear due to multi-mode vibrations of the support structure. In most cases, ordinary

rough methods which are based on the supports stiffness, damping, and mass input can't give a full picture of rotor-bearing-supports system vibrations. Some true resonance conditions produced by these supports might be out of proper attention. The difference between the values of critical speeds calculated for RBS against RBFS can be significant and can be reached from 10 to 30 percent [3].

To account for the structural support effect on rotor dynamics, several approaches can be used. A classical approach for rotor dynamics simulation is to model the support structure of a rotor-bearing system with a series of the journal bearing stiffness and damping and a spring-damper-mass system which represents support structure/casing mass, stiffness, and damping. In most cases, it is very difficult to estimate the mass, stiffness and damping characteristics of the rotor support.

One way to leverage these uncertainties is to perform equipment vibration measurements. Methods based on a dynamic model of the support structure creation from measured frequency responses is applied and discussed in [5], [7], and [8]. However, in many cases, especially at the rotor system design phase, experimental results are not available or it's undesirable from a cost-effectiveness point of view.

Another approach is to simulate a full conjugated model which includes detailed 3D FEA supports and rotor-bearing system models. The disadvantages of such an approach are that it results in an extremely complicated modeling process of the full system and long calculation time [9], [10].

An approach to reduce calculation time and overall complexity of FE models, based on modeling the rotor with solid elements and utilized transfer functions to represent the flexible support is proposed in [11]. A transfer function method is used to evaluate coupled rotor-foundation eigenfrequencies and to identify critical resonances in [12] and the comparison with measured data is discussed. A case study of a real shaft train is run based on MDOF approach including validation with experimental results [13]. In [14] the research is performed to study how turbomachines baseplates cause coupling transfer functions between bearings. Experimental validation of the developed approach is performed for the multistage centrifugal compressor.

The main goal of this article is to present a practical approach which allows one to account for structural support influence in terms of rotor dynamics response with high accuracy and effectiveness in relation to engineering efforts. This methodology was implemented in a rotor dynamics analysis software and is based on support vibrations modal reduction technics. As a preliminary step, the natural frequencies and their modal shapes are calculated for the separate support structure applying a three-dimensional FE model. Calculated relative displacements at the bearing location points and support's normalized modal characteristics (modal mass and modal stiffness) for each vibration mode is then imported in rotor dynamics beam-type FE model. The advantages of the current approach are the automation between all simulation steps, low calculation time and high accuracy of results which is confirmed by test examples.

1. METHODOLOGY DESCRIPTION

The equation of rotor-bearing system motion can be written in the following form:

$$M\ddot{x} + B\dot{x} + Kx = F(t) + R(t), \tag{1}$$
$$B = G + D, \tag{2}$$

where x – rotor displacement vector, $\dot{x}$ – rotor velocity vector, $\ddot{x}$ – rotor acceleration vector, M – mass matrix, G – gyroscopic matrix, D – damping matrix, K – stiffness matrix, $F(t)$ – excitation forces vector, $R(t)$ – bearing reactions vector for the rotor.

The equation of structural supports motion can be written as follows:

$$M_1\ddot{y} + D_1\dot{y} + K_1 y = R_1(t), \tag{3}$$

where $y, \dot{y}, \ddot{y}$ – support structure displacement, velocity and acceleration; M_1, D_1, K_1 – mass, damping and stiffness matrixes correspondingly; $R_1(t)$ – bearings reactions vector for supports.

Bearings reactions vector $R(t) = -R_1(t)$ can be expressed in terms of rotor and support structure displacement and velocity at bearings location as follows:

$$R(t) = K_B(x_B - y_B) + D_B(\dot{x}_B - \dot{y}_B), \tag{4}$$

where K_B equals bearing stiffness matrix, D_B is the bearing damping matrix, x_B, $\dot{x}_B$ is equal to rotor displacement and velocity at bearings location, and $y_B, \dot{y}_B$ represents structural supports displacement and velocity at bearings location.

Structural support displacements can be expanded as follows:

$$y(t) = \tilde{y}\alpha(t), \tag{5}$$

where $\tilde{y}$ is the matrix of foundation's eigenvectors, normalized by matrix of mass (or M-normal method), $\alpha(t)$ is the modal coefficients. Generally, the natural modes can be M-normalized (to the mass matrix), or K-normalized (to the stiffness matrix) – these two methods of normalization gives identical results.

The modal coefficients $\alpha(t)$ are satisfy following equations

$$\ddot{\alpha} + \delta\dot{\alpha} + p^2\alpha = R_1(t)\tilde{y}, \tag{6}$$

where δ is the unitary matrix of modal damping and p^2 is the unitary matrix of squared natural frequencies. At this step we assume that modal shapes are not coupled by resistance forces.

The major concern in rotor dynamics prediction – rotor forced vibrations which are the results of unbalance inertia forces (unbalance response). These forces can be presented in the following form:

$$F(t) = A\cos(\omega t) + B\sin(\omega t), \tag{7}$$

where A, B are the coefficients' vectors and ω is the angular rotor velocity.

Nodal displacement from equation (1) and modal coefficients from equation (5) can be found in the following form:

$$x(t) = C_1 cos(\omega t) + C_2 \sin(\omega t); \qquad (8)$$
$$\alpha(t) = C_3 cos(\omega t) + C_4 \sin(\omega t), \qquad (9)$$

where coefficients C_1, C_2, C_3, C_4 is determined from the system of linear algebraic equations.

Stability analysis is performed supposing the excitation forces are equal to zero and solution is written in the following form:

$$x(t) = Re\big(C_5 exp(\lambda t)\big); \qquad (10)$$
$$\alpha(t) = Re\big(C_6 exp(\lambda t)\big), \qquad (11)$$

where C_5, C_6 are eigenvectors and λ – eigenvalues. Analyzing the real part sign, we made a conclusion on system stability. If all eigenvalues have a negative real part, the motion is stable. If just one real part is positive, the motion is unstable.

Thus to describe structural supports, $(2 + N_B)N_M$ number of parameters, where N_B represents the number of bearings and N_M represents the number of supports' natural frequencies, should be determined.

Most state-of-the-art rotor dynamics software do use beam elements to model the shaft. Such an approach gives accurate analysis results (especially beams according to the Timoshenko formulation) and significantly speed-up calculation time. The proposed method for structural supports effect simulation described above allows for use of the beams elements for rotor dynamics analysis while structural supports/foundation) simulation is performed with 3D FE approach.

2. JEFFCOTT ROTOR-BEARING-STRUCTURAL SUPPORTS SYSTEM SIMULATION. METHODOLOGY VALIDATION

The Jeffcott rotor model with structural supports presented on the sketch in Fig. 1 is used for methodology validation.
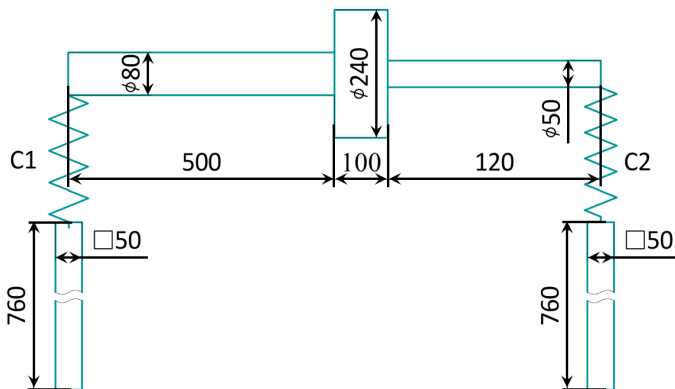

Figure 1: Jeffcott Rotor-Bearing Supports System

Isotropic bearings with stiffness's of C1=10E6 N/m and C2=10E7 N/m are used in the analysis.

To validate the proposed methodology, two identical rotor models were created in the rotor dynamics module (Fig. 2) and FE software (Fig. 3). Timoshenko theory is used to developed mechanical models for the rotor.
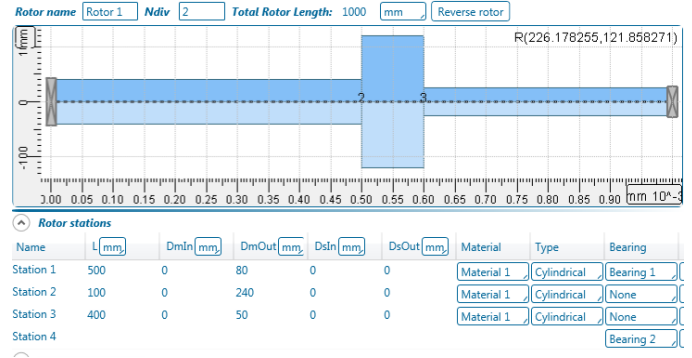

Name	L [mm]	DmIn [mm]	DmOut [mm]	DsIn [mm]	DsOut [mm]	Material	Type	Bearing
Station 1	500	0	80	0	0	Material 1	Cylindrical	Bearing 1
Station 2	100	0	240	0	0	Material 1	Cylindrical	None
Station 3	400	0	50	0	0	Material 1	Cylindrical	None
Station 4								Bearing 2

Figure 2: FE Model for Rotor-Bearing System without Structural Supports

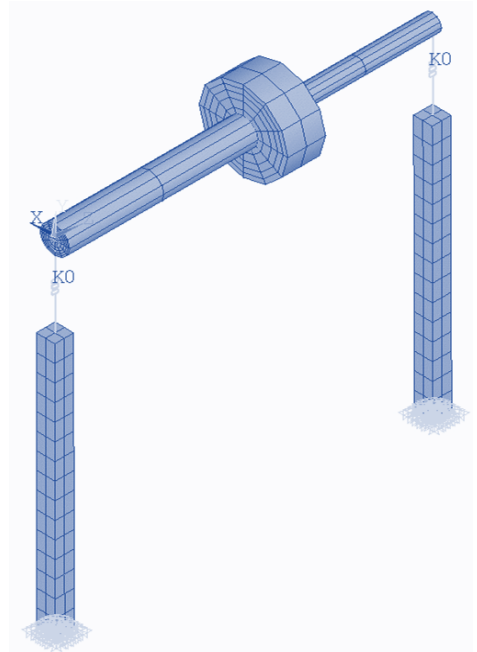The simple support structure was modeled using 3D solid 20-node hexahedral elements – see Fig. 3.

Figure 3: FE Model for Rotor-Bearing System with Structural Supports

According to the proposed methodology, at the first step, the separate supports structure was calculated and modal characteristics were used as the input data (Fig. 4) to simulate a rotor-bearing system response.

Foundation modal properties

Mass	Stiffness	Rotor/Station	X	Y
1	207641.3	Rotor 1.Station 4	435.15	0
1	207641.3	Rotor 1.Station 1	517.59	0
1	207641.3	Rotor 1.Station 4	-280.43	0
1	207641.3	Rotor 1.Station 1	9.8472	0
1	7848095.209	Rotor 1.Station 4	430.4	0
1	7848095.209	Rotor 1.Station 1	510.74	0
1	7848095.209	Rotor 1.Station 4	-274.71	0
1	7848095.209	Rotor 1.Station 1	-13.327	0
1	37818223.66	Rotor 1.Station 4	0	0
1	37818223.66	Rotor 1.Station 1	0	0
1	58076287.81	Rotor 1.Station 4	440.76	0
1	58076287.81	Rotor 1.Station 1	497.01	0
1	58076287.81	Rotor 1.Station 4	-237.75	0
1	58076287.81	Rotor 1.Station 1	-61.436	0
1	114863691.6	Rotor 1.Station 4	0	366.52
1	114863691.6	Rotor 1.Station 1	0	366.52
1	206548060.4	Rotor 1.Station 1	-274.79	0
		Rotor 1.Station 4	344.53	0
1	206548060.4	Rotor 1.Station 4	393.68	0
		Rotor 1.Station 4	243.16	0
1	206548060.4	Rotor 1.Station 1	86.689	0
		Rotor 1.Station 4	6.1295	0
1	206548060.4	Rotor 1.Station 1	6.4576	0
		Rotor 1.Station 4	-245.32	0

Figure 4: Supports Initial Data for Rotor Dynamics Analysis

Rotor natural frequencies for this case were calculated in the rotor dynamics module and the results are presented in Table 1. The conjugated rotor-bearing supports model was also simulated and the comparison of these two approaches is also presented in Table 1.

Table 1: Validation Results

Natural Freq. #	Modal Shape	Full RBFS FEA	Proposed Approach	Error, %
		Natural Frequency, Hz		
1	Rotor	10.192	10.191	0.009
2	Rotor	11.123	11.123	0.002
3	Rotor	34.865	34.859	0.018
4	Rotor	52.410	52.422	0.023
5,6	Support	72.564	72.509	0.076
7	Support	77.269	77.218	0.066
8	Rotor	108.907	108.906	0.001
9	Rotor	274.444	274.297	0.054
10	Rotor	275.363	275.237	0.046
11,12	Support	446.089	445.775	0.071
13	Support	446.832	446.518	0.07
14	Support	454.335	454.026	0.068
15	Rotor	665.086	663.721	0.206
16	Rotor	665.153	663.790	0.205
17,18	Support	979.244	978.552	0.071
19	Support	1213.540	1212.642	0.074
20	Support	1213.8	1212.642	0.095
21	Support	1216.14	1213.401	0.226

Comparison of the results illustrated in the table shows good agreement between proposed methodology and the full FE rotor-bearing supports model since the results deviation does not exceed 0.23%.

3. SINGLE STAGE COMPRESSOR ROTOR-BEARING STRUCTURAL SUPPORT SYSTEM SIMULATION

The proposed methodology was applied to simulate rotor dynamics for a newly designed single stage compressor with the structural support presented in Fig. 5.
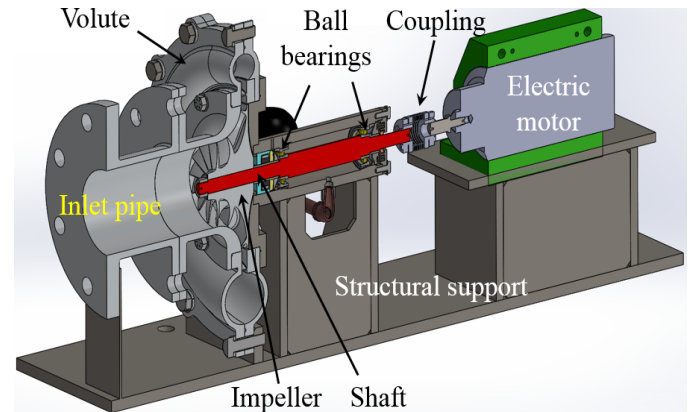
Figure 5: Single Stage Compressor 3D CAD Assembly

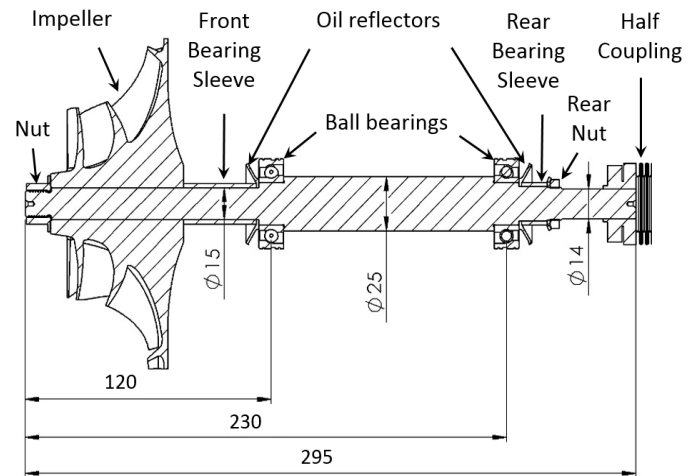The compressor rotor rated speed is 60000 RPM. The rotor assembly is presented in Fig. 6.

Figure 6: Rotor Assembly

Mass and inertia data for the rotor components are listed in Table 2.

The structural support model consists of a volute casing with additional pipes for performance measurement equipment setup, pedestals for compressor rotor and housing, the pedestal and housing for electric motor and motor with coupling.

4

Table 2: Rotor Mass-Inertia Characteristics

Component	Mass, kg	Diametric Inertia, kg·m²
Nut	0.02	0.000001
Impeller	0.63	0.00058
Front bearing sleeve	0.03	0.0000045
Rear bearing sleeve	0.012	0.0000008
Ball bearing inner ring	0.02	0.0000015
Oil reflector	0.01	0.0000011
Rear Nut	0.008	0.0000004
Half coupling	0.038	0.0000008

At the first step of the work, modal analysis for supports structure is performed. Tetrahedral solid FE with 10 nodes and three DOF at each node are used to create FE model. The motor with half coupling has been included in the FE model as an equivalent solid cylinder with the same mass/center mass. FE mechanical model is presented below in Fig. 7.
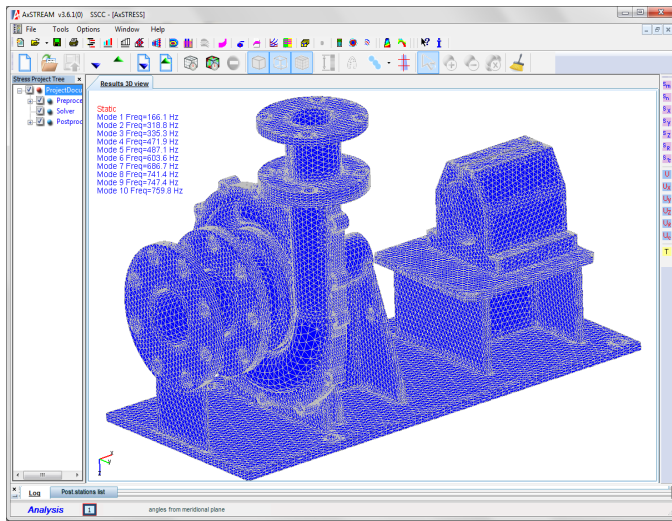

Figure 7: Supports Structure FE Model

The equation for natural frequencies calculation is presented in the general form below:

$$\det([K] - \omega^2[M]) = 0, \qquad (12)$$

where [K] is the stiffness matrix and [M] is the mass matrix.

The mode shape and natural frequencies for modes with the lowest frequencies are presented in Fig. 8. Structural support modal analysis results (natural frequencies and corresponding relative displacements along the X, Y and Z axis' on bearing locations) are then imported into the rotor dynamics module to account for the effect of supports vibration on rotor response. The construction damping coefficient for the supports structure was assumed to be equal 0.02.
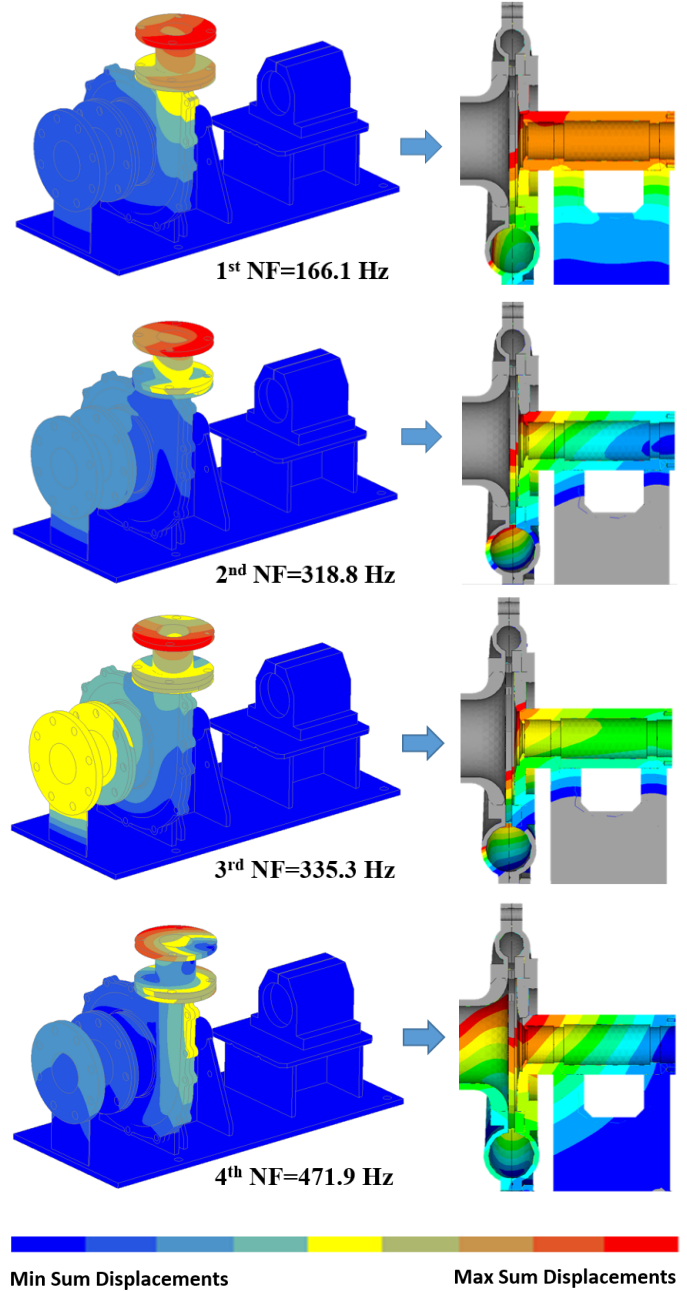

1st NF=166.1 Hz

2nd NF=318.8 Hz

3rd NF=335.3 Hz

4th NF=471.9 Hz

Min Sum Displacements   Max Sum Displacements

Figure 8: **Structural Support Modal Shapes Corresponding to Lowest Natural Frequencies**

A rotor dynamics model for the single stage centrifugal compressor was created in the RotorDynamics software and presented in Fig. 9. Ball bearings stiffness was calculated in the Bearing module according to methodology described in [15] (approach assumes the same stiffness in horizontal and vertical directions and cross-coupling stiffness is assumed to be equal to zero). Ball bearing damping was estimated according to recommendations presented in [3].

Lateral rotor dynamics analyses are performed based on Timoshenko theory.
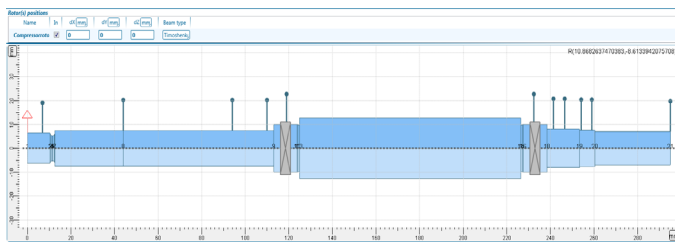
Figure 9: **Centrifugal Compressor Rotor Dynamic Model**

Critical speeds analysis was performed on the rotor without supports. Critical speeds and corresponding modal shapes are presented in Fig. 10 for the analysis with rigid bearings. The analysis results for the rotor with elastic bearings are shown in Fig. 11.
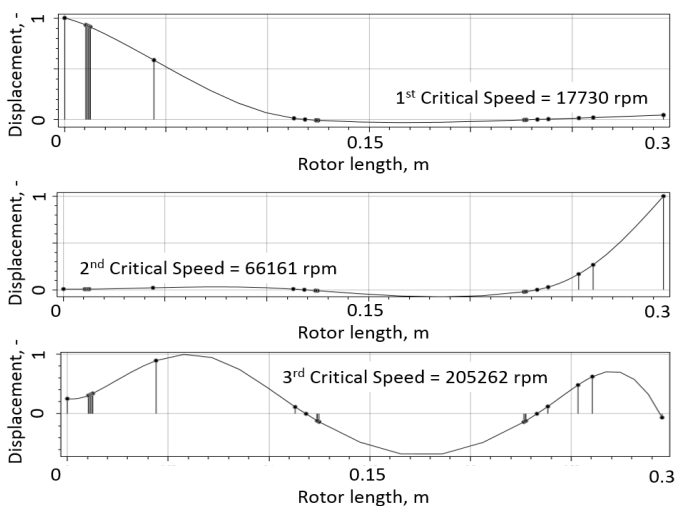

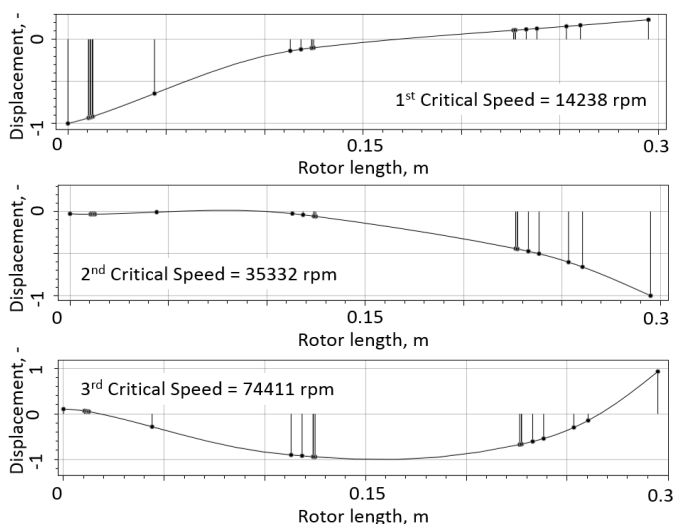**Figure 10**: **Critical Speeds and Modal Shapes for the Rotor on Rigid Supports**

Figure 11: **Critical Speeds and Modal Shapes for the Rotor on Elastic Supports**

Campbell diagram for the Compressor Rotor is shown in Fig. 12
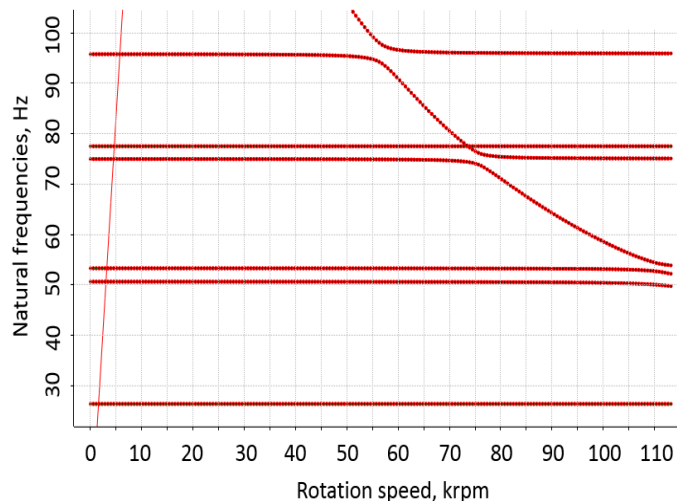

Figure 12: **Campbell Diagram**

To investigate the supports influence on rotor response, unbalance response analyses were performed for two cases:
1) for the rotor-bearing system without support;
2) for the rotor-bearing system with support structures.

Following [4], unbalances were placed at left rotor end (Impeller side) and right rotor end (Coupling side) to excite mode shape corresponding 1st and 2nd critical speed as shown in Fig. 13, where red arrows represent unbalance.
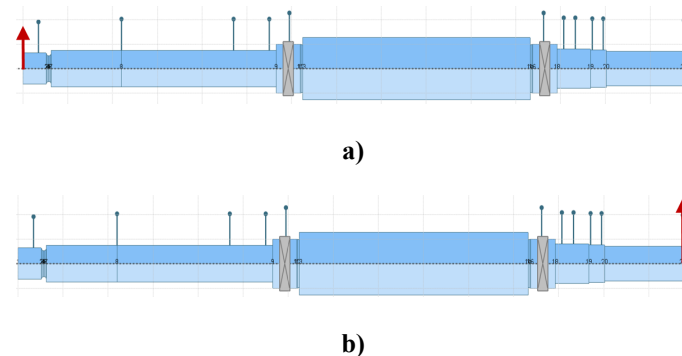

a)

b)

Figure 13: **Unbalances to Excite Mode Shape Corresponding: (a) 1st Critical Speed; (b) 2nd Critical Speed**

The unbalance values were calculated according to [4] and are equal to 0.089 g-mm to excite the mode corresponding to 1st critical speed and 0.023 g-mm to excite the mode corresponding to 2nd critical speed

Unbalance response simulation results (maximum vibration amplitudes) for the rotor-bearing system without structural supports are presented in Fig. 14. A simulation was performed for the rotor speed from zero to 125% of the trip speed.
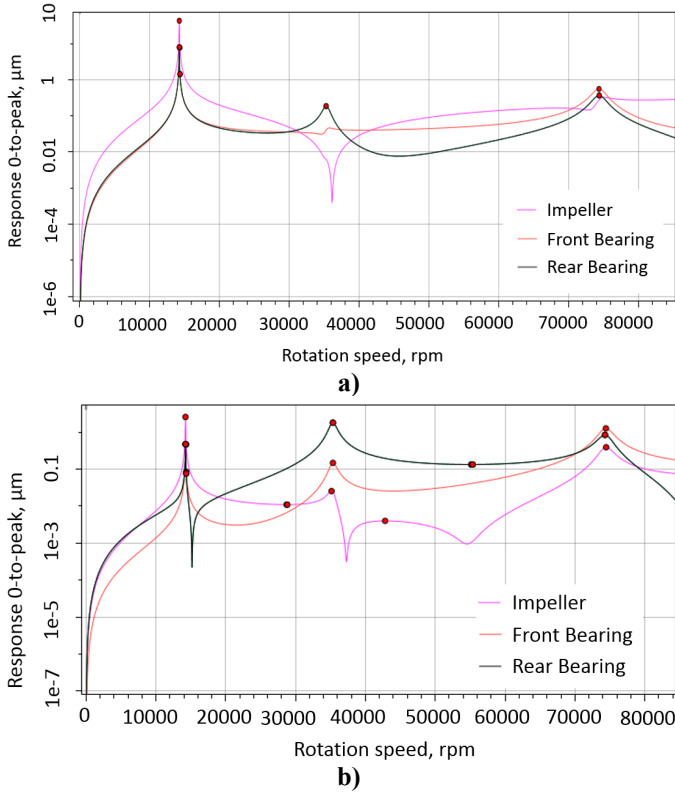
Figure 14: Frequency Response Characteristics for Rotor-Bearing System without Supports Measured at (a) Impeller, (b) Front Bearing, (c) Rear Bearing Locations

Bearing dynamical forces in horizontal and vertical directions at the front and rear bearings at corresponding resonance points and nominal operation for compressor rotor-bearing system without structural supports are presented in Table 3.

Table 3: Resonances Analysis for Rotor-Bearing System without Structural Supports

Nominal / Resonance Speed, rpm		14238	35277	74360	60000
	Direction	Dynamic Force, N			
Unbalance placed to excite 1st mode shape					
Front Bearing	Horizontal	148.3	0.76	10.5	1.1
	Vertical	148.3	0.76	10.5	1.1
Rear Bearing	Horizontal	66.16	1.55	3.05	0.18
	Vertical	66.16	1.55	3.05	0.18
Unbalance placed to excite 2nd mode shape					
Front Bearing	Horizontal	8.96	2.78	24.02	1.15
	Vertical	8.96	2.78	24.02	1.15
Rear Bearing	Horizontal	3.97	15.08	7.08	1.18
	Vertical	3.97	15.08	7.08	1.18

The dynamic forces at the front and rear bearings at nominal operation are less than 1.18 N which is acceptable.

The number of modes, which should be used in the analysis, can be optimized [13]. A sensitivity study was performed for the full system which includes the rotor-bearing system and supports in order to find the modal shapes number sufficient to provide accurate results. The unbalance response was calculated with considerations of different numbers of supports' modal shapes. The comparison between 10, 35, 45 and 50 supports' modes involved into rotor dynamics analysis is illustrated in Fig. 15, where the amplitudes were measured at the impeller location.

The sensitivity study shows that difference between frequency-response characteristics is noticeable up to the case when 45 modes were considered. Further expansion of modes number provides almost the same the same results (error is less than 0.1%). So, the sensitivity study shows that 45 mode shapes with lowest frequencies should be considered in the analysis.
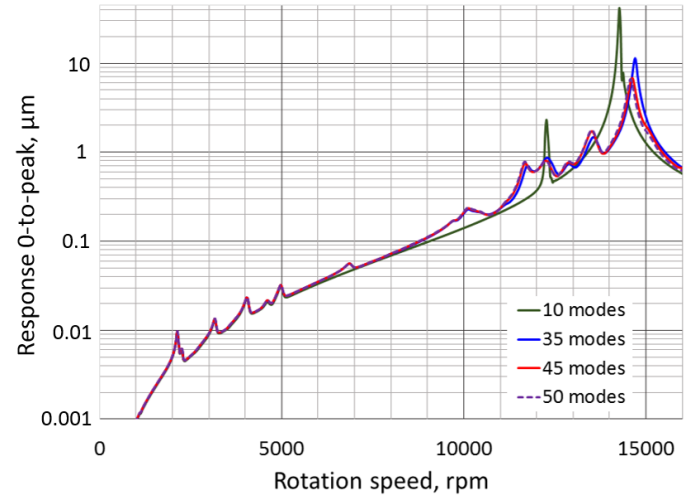

Figure 15: **Sensitivity Study Results: (a) Horizontal Direction Amplitude, (b) Vertical Direction Amplitude, (c) Maximal Amplitude**

Frequency-response characteristics for the rotor-bearing system with structural supports are presented in Fig. 16, where maximal vibration amplitudes are shown at critical rotor locations (impeller, front and rear bearing). These results were calculated taking in the account 45 supports' modal shapes.

The comparison for RBS and RBFS response is presented in Fig. 17 for the case when the unbalance is placed to excite mode shape corresponds to 1st critical speed.

It can be observed that the foundation generates several additional resonances in the system. Not all of the support's modal shapes produce the resonance peaks. Different support mode shapes influence the rotor response in a different way. The amplitude values of the peaks are also different and are lower in the case of RBFS, because of the additional support structure damping.

The RBS has the same response amplitudes in both horizontal and vertical directions because of isotropic bearings (red line in Fig. 15), while the responses for RBFS are different in horizontal and vertical directions (green and violet dotted lines in Fig. 15). This can be explained by the anisotropy of structural support's properties.
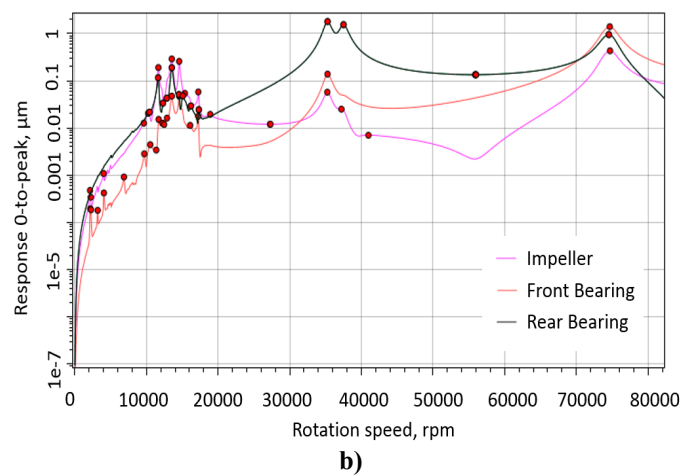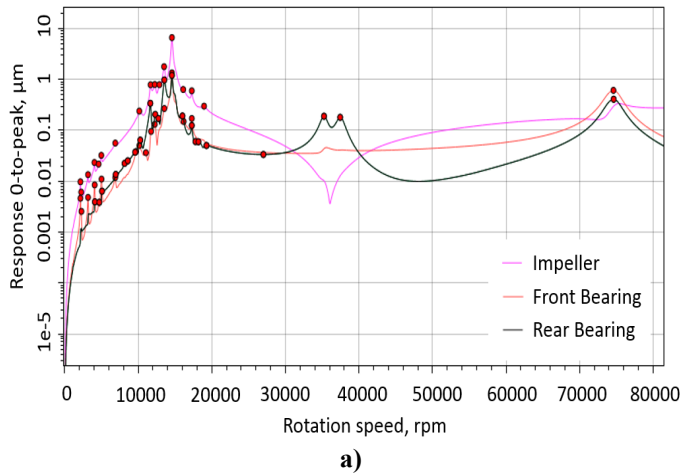
Figure 16: Frequency-response characteristics for RBFS. Unbalance applied to excite the mode corresponding to (a) 1st critical speed (b) 2nd critical speed
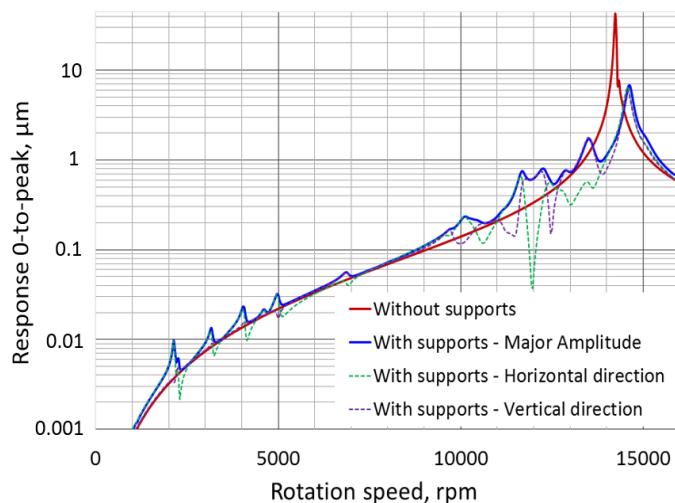

Figure 17: **The Comparison for RBS and RBFS Response**

Bearing dynamical forces for compressor rotor-bearing system with structural supports are presented in Table 4.

Table 4: Resonances Analysis for Rotor-Bearing System with Structural Supports

Nominal / Resonance Speed, rpm		14571	35246	74600	60000
	Direction	Dynamic Force, N			
Unbalance placed to excite 1st mode shape					
Front Bearing	Horizontal	25.10	0.67	10.28	0.17
	Vertical	19.44	0.75	8.95	0.18
Rear Bearing	Horizontal	5.52	0.43	3.04	0.17
	Vertical	8.92	1.53	2.62	0.18
Unbalance placed to excite 2nd mode shape					
Front Bearing	Horizontal	0.83	0.23	23.20	0.99
	Vertical	0.79	2.59	20.70	1.15
Rear Bearing	Horizontal	0.33	2.24	7.19	1.16
	Vertical	0.34	14.80	6.00	1.17

The dynamic forces at the front and rear bearings at nominal operation are less than 1.17 N. The bearing DF for RBFS are decreased at resonance conditions in comparison to the RBS.

CONCLUSION

Structural supports influence rotor response and should be considered in the rotor dynamics analysis process at the design step. An improved automated approach for the structural support's influence predictions on rotor dynamics is proposed. The main advantage of the methodology is automation and integration between all analysis steps within the software modules. Such approach accelerates a rotor-bearing-support system dynamics analysis:

- calculation time decreases in 5-7 times in comparison with full 3D models analysis;
- the modeling process is easier which also saves time.

The validation of the methodology is done based on the Jeffcott rotor model on simple solid structural supports and shows high accuracy in the results (the error is less than 0.3 percent).

The method was applied to single stage compressor lateral rotor dynamics, and influence of structural supports on rotor response have been studied including:

- supports mode shape number;
- effect of supports stiffness anisotropy in vertical and horizontal directions;
- supports influence on bearings dynamical forces.

The single stage compressor, described in the paper, is in the final phase of commissioning and in the future, it's planned to be tested to prove analysis methodology by experimental validation.

The method is recommended to be applied to many industrial applications (turbine, compressors, fans, pumps, and etc.), where rotating equipment is supported by pedestals, tabletop-type foundation, frame and pipe supports of arbitrary geometry supports.

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